Horizontal projection

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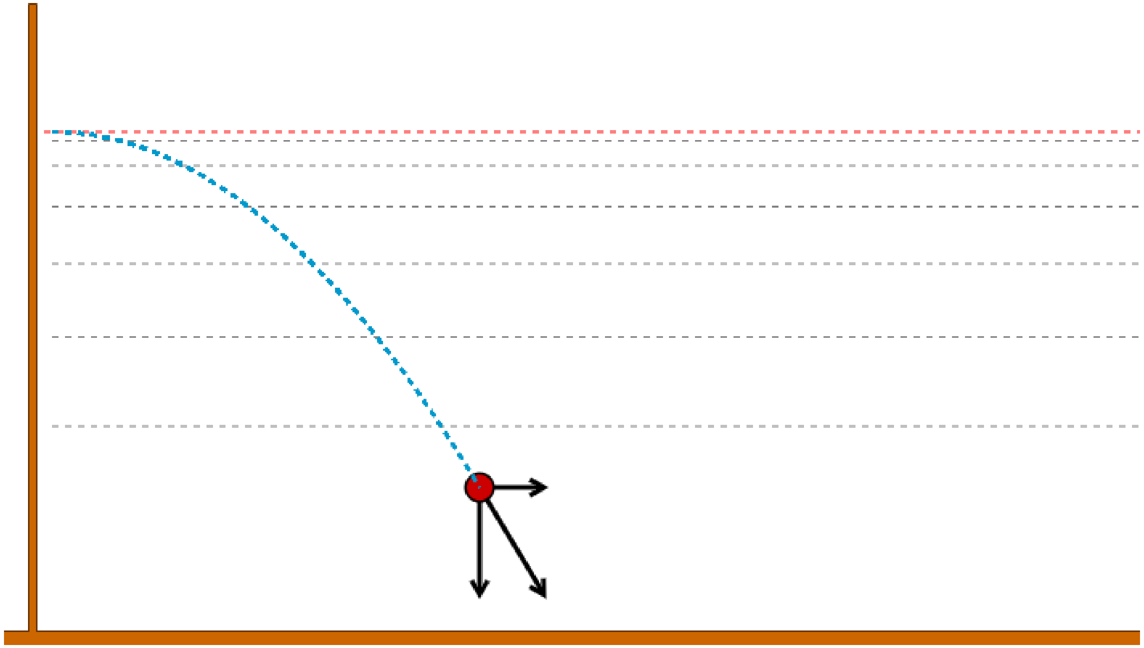
Horizontal projection from the top of a tower



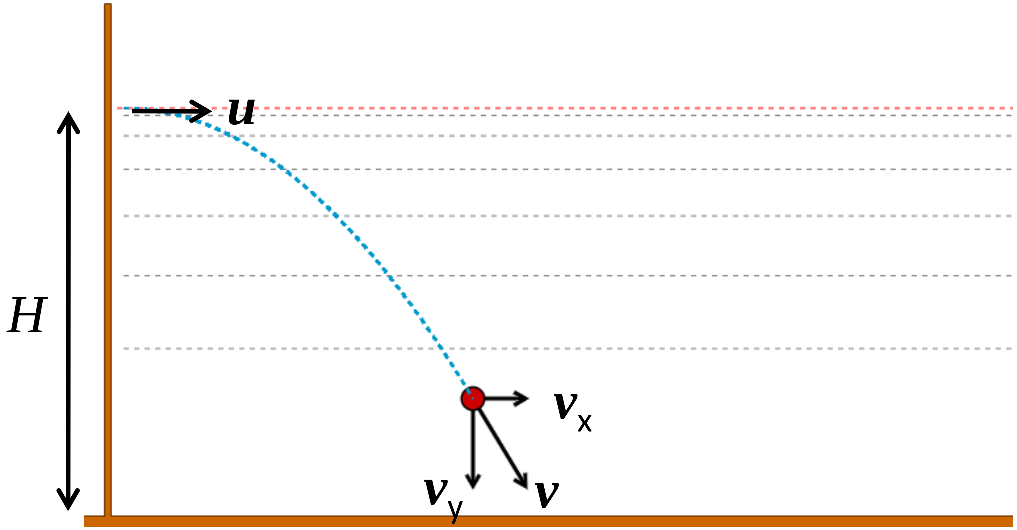
Time of descent (TOD)

Range (R)

Final velocity (v)

Path of projectile

Horizontal projection from the top of a tower



Click here for simulation

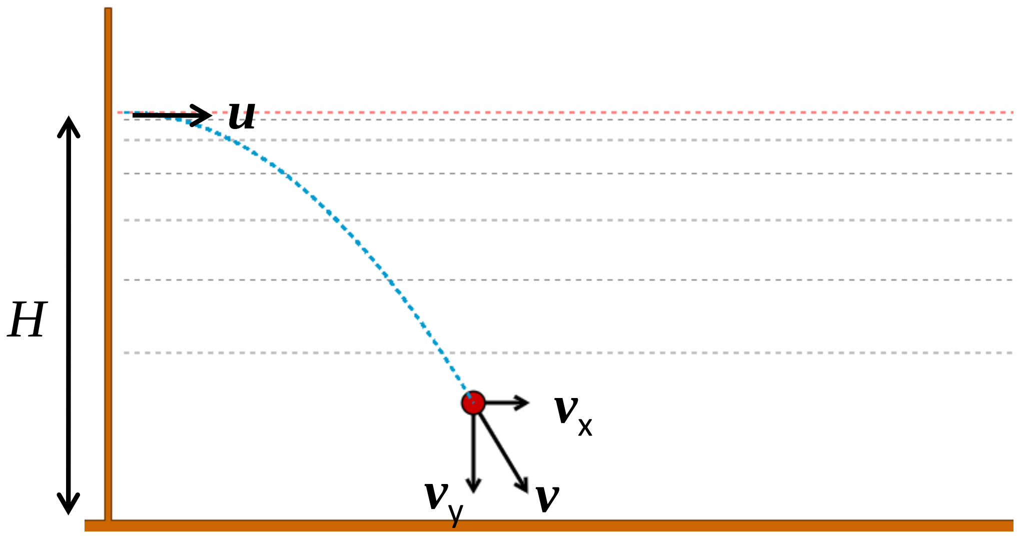
A body is projected horizontally with initial velocity $\boldsymbol{u}$ from the top of a tower of height H.

Initial velocity of the body is only in the horizontal direction and acceleration (due to gravity is only in the vertically downward direction. Therefore

$$u_x = \mathrm{u} \text{ and } u_y = 0$$
$$a_x = 0 \text{ and } a_y = \mathrm{g}$$

Horizontal component of velocity remains constant. Vertical component of velocity increases in the downward direction during descent (because of g) .

Horizontal projection from the top of a tower



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Time of descent

Considering only the vertical component of the motion, for a body moving with constant acceleration, displacement is given by

$$S = ut + \frac{1}{2}at^2$$

Using u = 0 and a = - g (as it is downwards) and S = -H (also downwards) we get

$$-H = \frac{1}{2}(-g)t^2$$

$$TOD = \sqrt{\frac{2H}{g}}$$

Note that this is same as the time of descent of a freely falling body !
The initial velocity in horizontal direction, given to the projectile has no effect of the TOD.

Range (R)

$$S = ut + \frac{1}{2}at^2$$

$$S_x = u_x t + \frac{1}{2}a_x t^2$$

There is no acceleration in the horizontal direction, therefore

$$x = u_x\ t + 0$$

When the time interval is equal to the time of descent then the horizontal displacement of the body is equal to its range therefore

$$R = u\sqrt{\frac{2H}{g}}$$

Final velocity (v)

Final velocity of the body consists of both horizontal and vertical components.
Horizontal component of velocity remains constant and vertical component increases with time. Therefore

$$v_x = u + 0 \quad \boxed{\text{i}}$$

$$v_y = 0 + gt$$

When the time interval is equal to the time of descent then the vertical component of velocity becomes

$$v_y = 0 + g\sqrt{\frac{2H}{g}}$$

$$v_y = \sqrt{2gH} \quad \boxed{\text{ii}}$$

Using equation (i) and (ii) we get

$$\boldsymbol{v} = u\hat{i} + \sqrt{2gH}\ \hat{j}$$

Angle (measured clockwise) made by the velocity vector w.r.t. the ground is given by

$$\tan(\theta) = \frac{\sqrt{2gH}}{u}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{2gH}}{u}\right)$$

Final speed is given by the magnitude of velocity i.e.

$$v = \sqrt{u^2 + 2gH}$$

Path of projectile

Horizontal component of displacement is given by the relation

$$x = ut$$

Therefore

$$t = \frac{x}{u} \quad \text{--- } \boxed{\text{i}}$$

Vertical component of displacement is given by

$$y = 0 - \frac{1}{2}gt^2 \quad \text{--- } \boxed{\text{ii}}$$

Using equation (i) in the above equation we get

$$y = \frac{1}{2}g\left(\frac{x}{u}\right)^2$$

This is an equation of a parabola. Therefore the body follows a parabolic path.